XXVIII. *Of Triangles described in Circles and about them.* *By* John Stedman, *M. D.*

Redde, March 23, 1775.

PROPOSITION I.

An equilateral triangle inscribed within a circle is larger than any other triangle that can be inscribed within the same circle.

LET ABC (TAB. VIII. fig. 1.) be an equilateral triangle, inscribed in the circle ADCB; and let ADE be a triangle supposed larger than ABC. Let ADE be drawn with one of its angles at the same point with one of the angles of the equilateral triangle, suppose at A, and then its other two angles will fall either on the segments ADB and AEC, or one of the angles will on the segment BC. First, let one of its angles fall at D, between A and B; and the other at E, between A and C, and draw the line BE. In the triangles ABC, ABE, the triangle ABF is common, and the two remaining triangles BFC, AFE, are similar; for the angle AFE is equal to its opposite angle BFC; and the two angles EAC, EBC, are equal, being subtended by the same segment EC, and so the two remaining angles

AEF,

AEF, BCF, muſt be equal; wherefore the ſides are proportional, and BC and AE, ſubtending equal angles muſt be homologous; but BC is equal to AC, which is greater than AE; conſequently the triangle BFC is greater than AFE, and ſo the equilateral triangle ABC is greater than the triangle ABE. In the ſame manner, the triangle ABE may be proved greater than ADE; for AHE is common, and the two triangles ADH, BHE, are ſimilar, and their ſides proportional; and AD and BE, ſubtending equal angles, muſt be homologous; but BE is greater than BC, which is equal to AB, and that again greater than AD; conſequently BE is greater than AD, and the whole triangle AEB greater than AED; and ſo the equilateral triangle muſt, *à fortiori*, be greater than AED. *Q. E. D.*

Next, let the triangle ADE be ſuppoſed greater than the equilateral triangle ABC, and let the angle ADE fall ſomewhere in the ſegment BDC, (ſee fig. 2.) ſo that the ſegment EC may be greater than BD; for if it were not, the angle AED being applied to any of the angles of the equilateral triangle, the demonſtration would become the ſame as in the firſt caſe: wherefore the ſegments AEC, BDC, being equal, and BD being leſs than EC, AE muſt be leſs than DC. Draw the right line DC, and then in the two triangles ADC, ADE, the triangle AFD is common, and the two triangles AFE, DFC, are equiangular and ſimilar, and the ſides AE, DC, ſubtending equal angles, are homologous; but DC is greater than AE; ſo the triangle DFC is greater than the triangle AFE, and the whole

 triangle

triangle ADC is greater than ADE; but the equilateral triangle may be proved greater than ADC from the first case, and consequently greater than ADE. *Q. E. D.*

PROPOSITION II.

An equilateral triangle described about a circle is less than any other triangle that can be described about the same circle. Fig. 3.

LET the equilateral triangle ABC be described about the circle HIK, and let the triangle BDG be supposed less than the equilateral triangle. Draw the line AF parallel to BC, then the triangles AFE, EGC, are similar; for the opposite angles AEF, GEC, are equal, as likewise the angle AFE to the angle EGC; the lines AF and GC being parallel, and falling upon the same line FG, the angles AFE and EGC are therefore equal, and the sides AE, EC, subtending equal angles, are homologous; but the side of the equilateral triangle AC being equally divided at I, the line AE is greater than EC, and consequently the triangle AFE is larger than the triangle EGC; and the triangle DAE much larger than EGC: therefore, in the triangles DBG and ABC the part ABGE being common the whole triangle DBG is larger than the equilateral triangle. *Q. E. D.*

Whatever other triangles can be described about a circle, may be demonstrated to be larger than an equilateral triangle described about the same circle, upon the same principles as the preceding.

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PROPOSITION III.

The square of the side of an equilateral triangle inscribed in a circle is equal to a rectangle under the diameter of the circle, and a perpendicular let fall from any angle of the triangle upon the opposite side. Fig. 4.

THE two triangles ADC, AEC, are equi-angular and similar, the angles ACD, AEC, being both right, and that at A common; therefore AD : AC :: AC : AE, and $AC^2 = AD \times AE$. *Q.E.D*(a).

The square of one side of the triangle being compleated so as to include the triangle, I say, that part of the side of the square that falls within the circle is equal to the radius; and the other part, lying without the circle, is equal to the radius *minus* twice the portion lying between the side of the square, and the circumference of the circle; or is equal to that part of the radius that lies between the centre and the side of the square *minus* the remainder of the radius; that is CL is

(*a*) And universally, a perpendicular being drawn from any angle of a right-lined triangle to the opposite side, the rectangle under the two sides which contain that angle, is equal to the rectangle under the perpendicular and the diameter of the circumscribed circle. (See TAB. VIII. fig. 5.)

From A, one angle of the triangle BAC, draw AE perpendicular to the side BC. Round the triangle BAC describe a circle, and draw the diameter AD. I say, the rectangles $AC \times AB$, $AE \times AD$, are equal. Join DB. The angle DBA is a right angle. Therefore it is equal to the right angle CEA. The angles at the circumference ACE, ADB, are equal, because they stand upon the same arc AB. Therefore the two triangles ACE, ADB, are similar, and AC : AE = AD : AB. Therefore $AC \times AB = AE \times AD$. *Q. E. D.* S. HORSLEY.

is equal to the radius, and LI=KG—2MG; or LI=KM−MG. FG being parallel to BC, and confequently perpendicular to IC, muft divide the chord LC in two equal parts; fo that MC being equal to KE, LC muft be equal to 2KE; but KE (by EUCL. l. XIII. pr. 12. cor. 2. Clav.) is equal to ED; therefore LC = KD the radius. The fide of the fquare IC being equal to BC is likewife equal to NM; but LC being equal to KG, the remaining part LI muft be equal to NK − MG; or to KM—MG. *Q. E. D.*

Fig. 1.

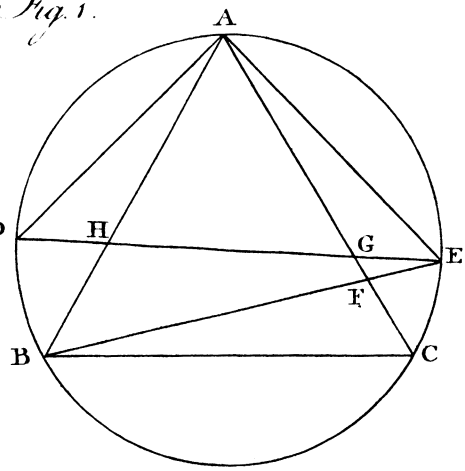


Fig. 3.

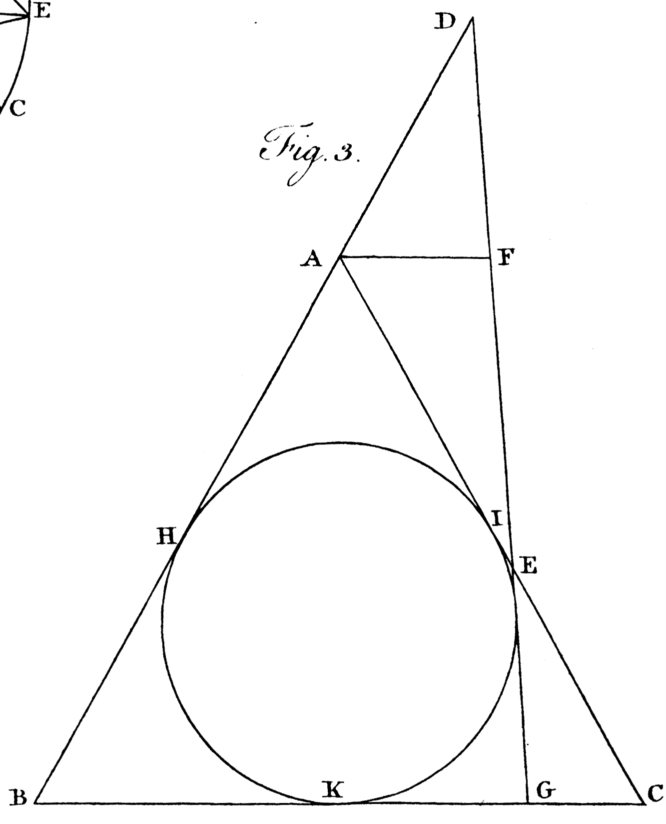


Fig. 5.

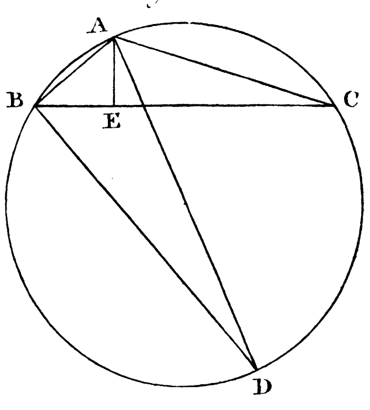


Fig. 2.

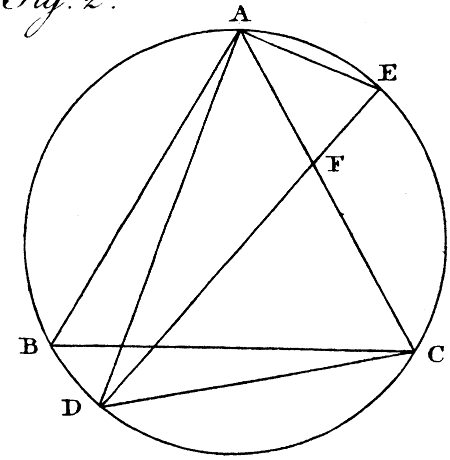
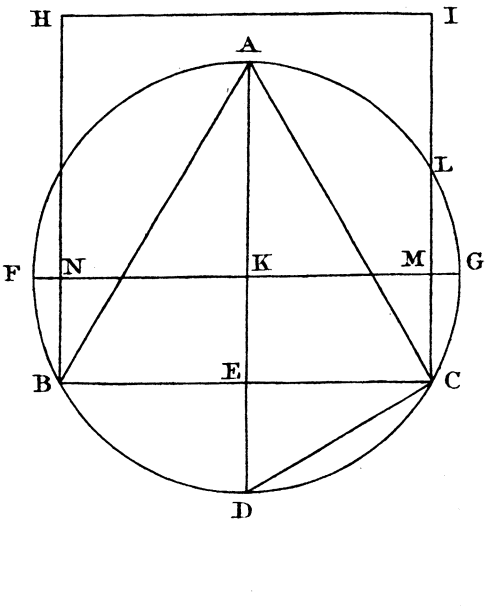


Fig. 4.



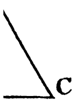


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Fig. 1.

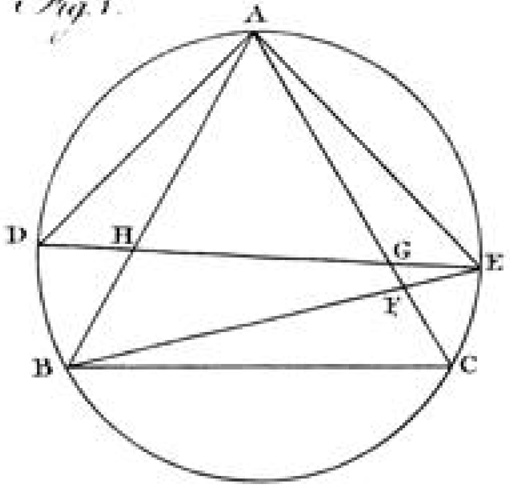


Fig. 2.

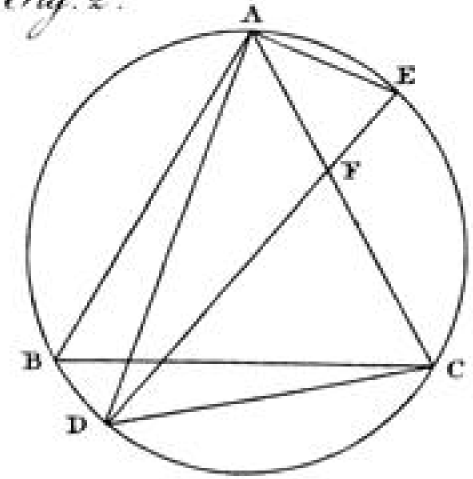
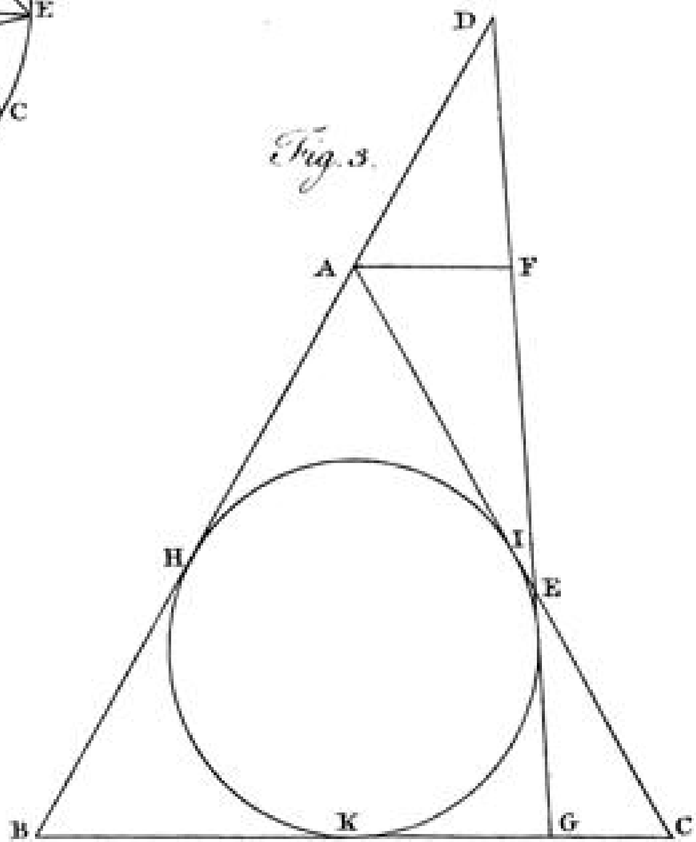


Fig. 3.



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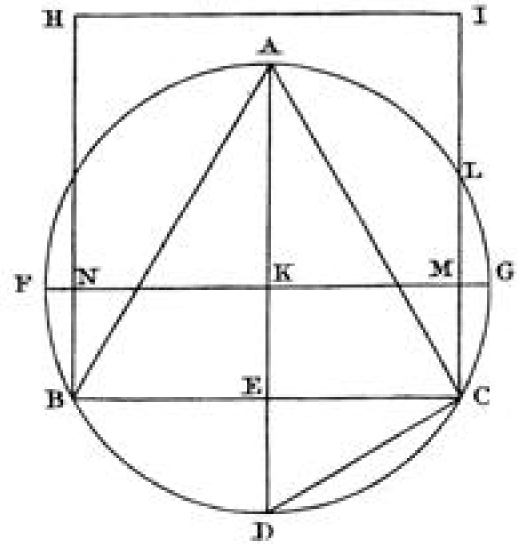

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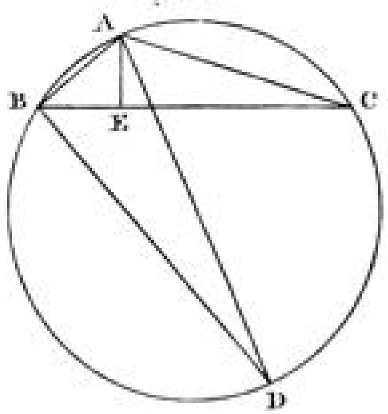

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